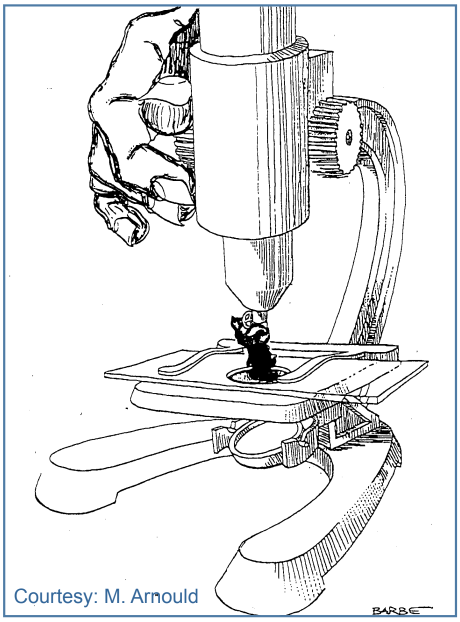
Experimental Nuclear Astrophysics:
An Introduction

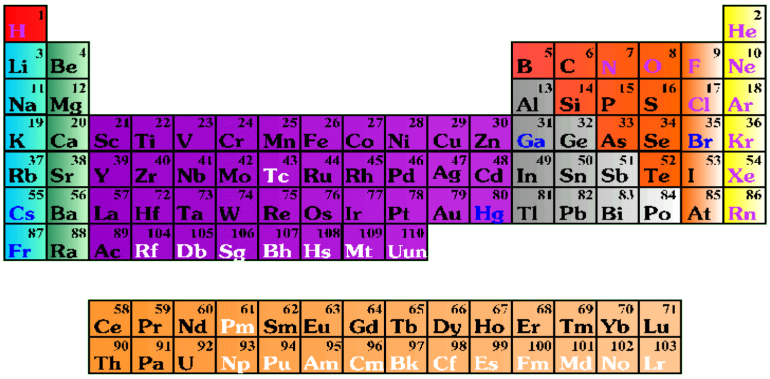


Courtesy: M. Arnould

Marialuisa Aliotta
School of Physics and Astronomy - University of Edinburgh
Scottish Physics Universities Alliance

(EXPERIMENTAL) NUCLEAR ASTROPHYSICS

- study energy generation processes in stars
- study nucleosynthesis of the elements



- What is the origin of the elements?
- How do stars/galaxies form and evolve?
- What powers the stars?
- How old is the universe?
- …

NUCLEAR PHYSICS

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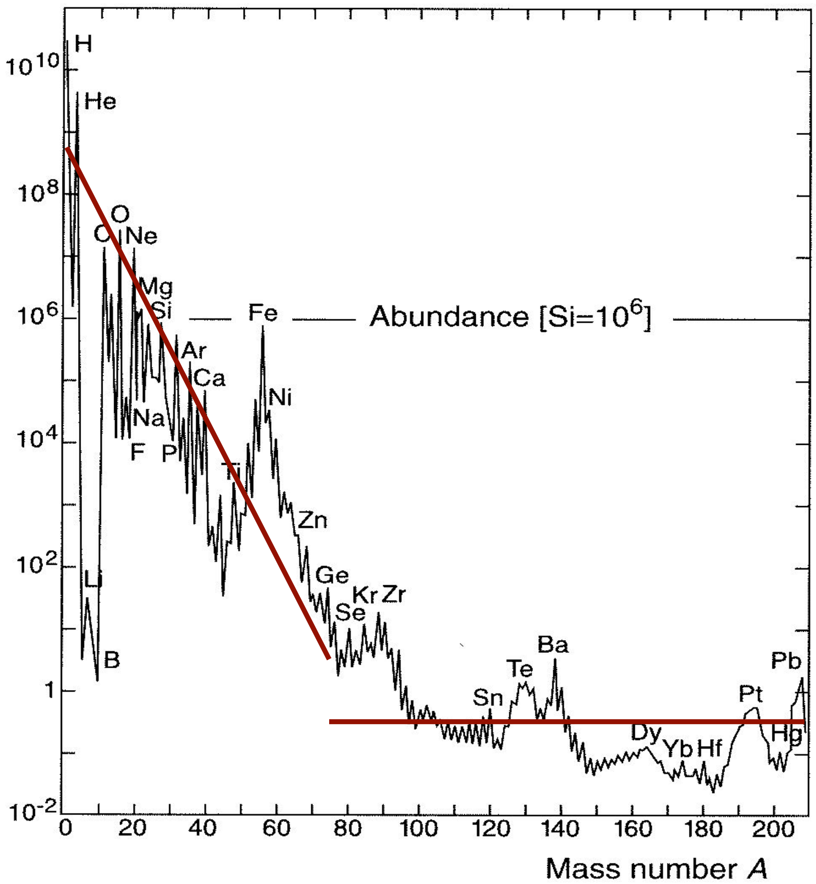
KEY for understanding

MACRO-COSMOS intimately related to MICRO-COSMOS

Abundance curve of the elements

Cosmic (Standard) Abundance Curve (SAD)



Data sources:

Earth, Moon, meteorites, cosmic rays, solar & stellar spectra…

Features:

- distribution everywhere similar
- 12 orders-of-magnitude span
- H ~ 75%, He ~ 23%
- C → U ~ 2% ("metals")
- D, Li, Be, B under-abundant
- exponential decrease up to Fe
- almost flat distribution beyond Fe

Why these feature?

REVIEWS OF

MODERN PHYSICS

VOLUME 29, NUMBER 4 OCTOBER, 1957

Synthesis of the Elements in Stars*

E. MARGARET BURBIDGE, G. R. BURBIDGE, WILLIAM A. FOWLER, AND F. HOYLE

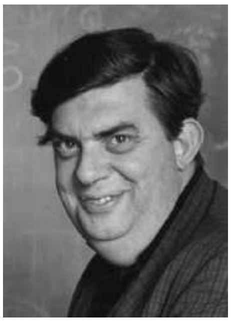
Kellogg Radiation Laboratory, California Institute of Technology, and Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology, Pasadena, California

Rev. Mod. Phys. 29 (1957) 547

(B^2FH, 1957)

Burbidge

Burbidge



Fowler

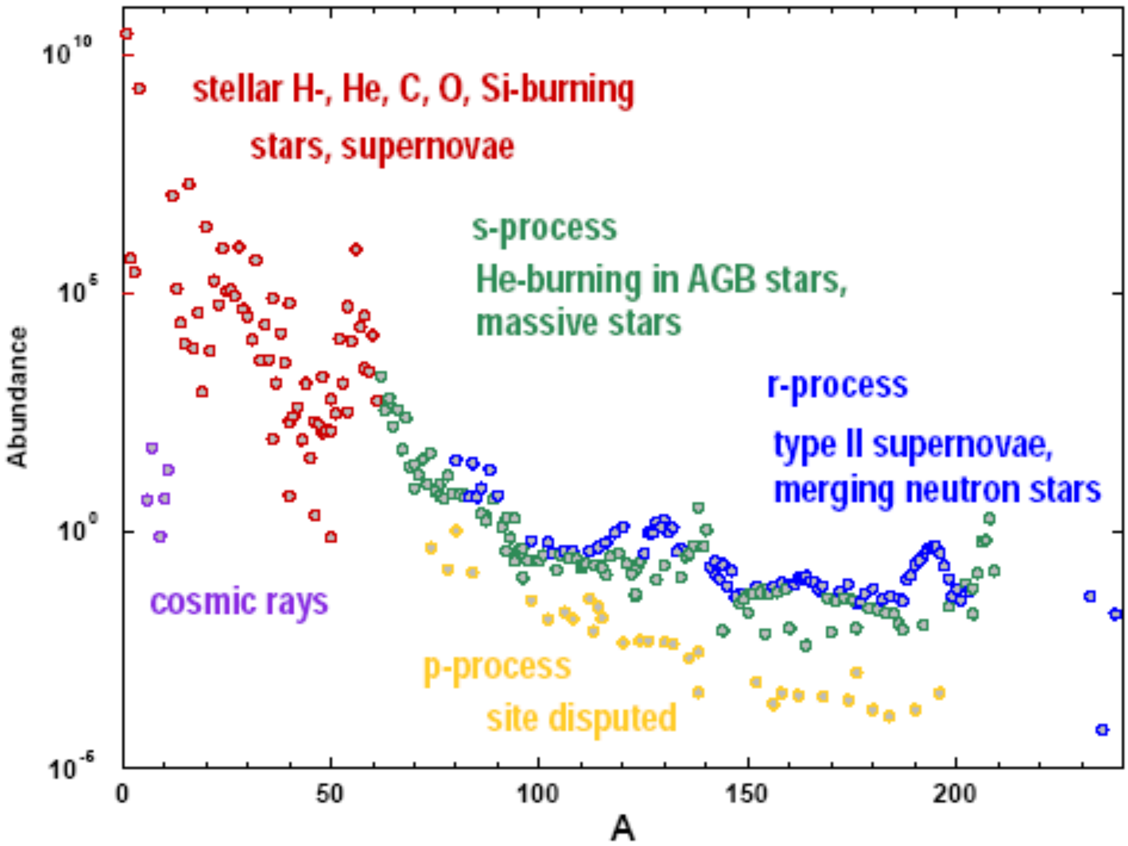
Hoyle

1983
Nobel Prize

"for his theoretical and experimental studies of the nuclear reactions of importance in the formation of the chemical elements in the universe"

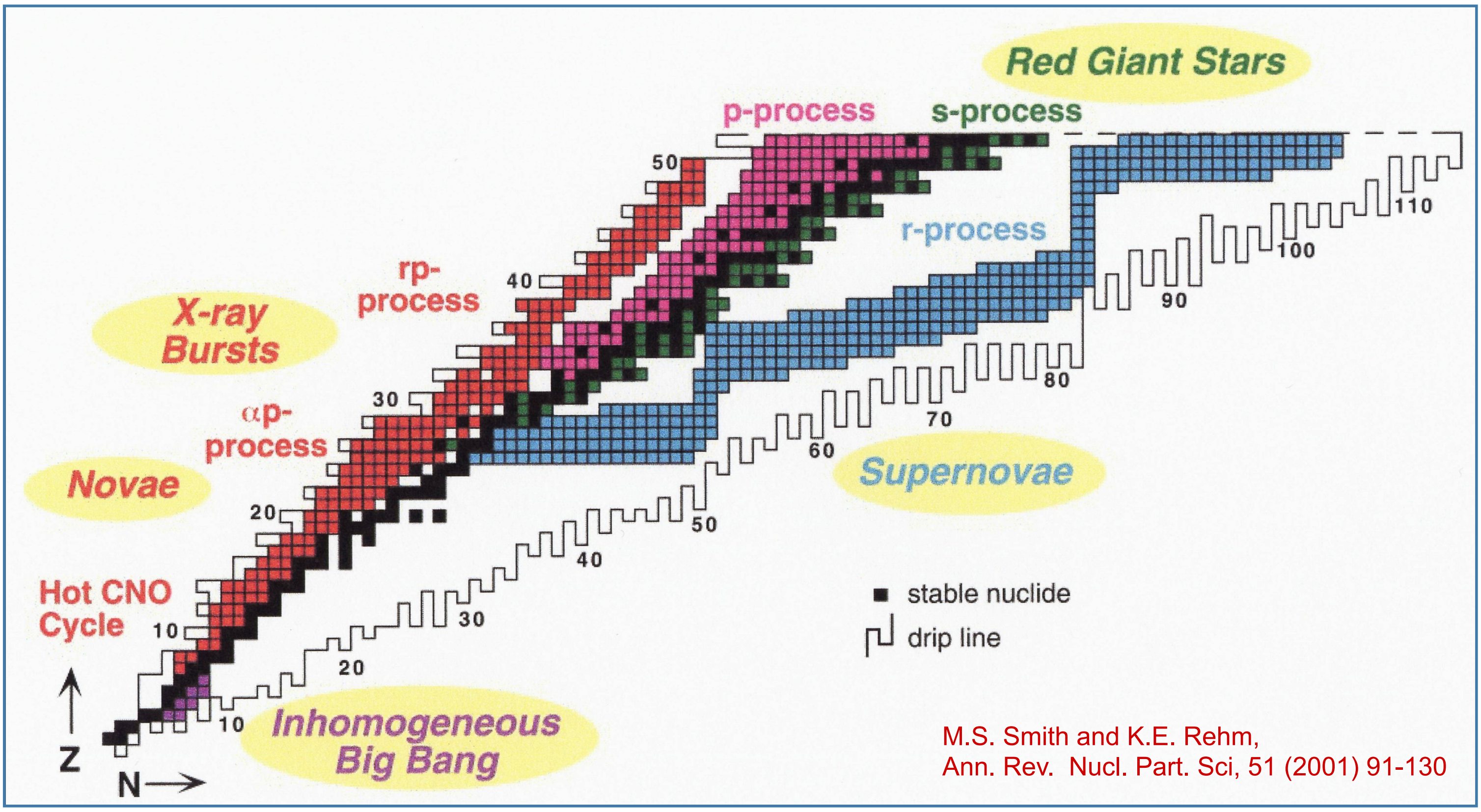
The Synthesis of the Elements in Stars

Burbidge, Burbidge, Fowler & Hoyle (B^2FH): Rev. Mod. Phys. 29 (1957) 547



from: M. Wiescher, JINA lectures on Nuclear Astrophysics

Overview of main astrophysical processes



the vast majority of reactions encountered in these processes involve UNSTABLE species
hence the need for **Radioactive Ion Beams**

Lectures' layout

- **introduction**
  - the tools of the trade
  - non-resonant cross section
  - resonant cross section
  - stellar reaction rates

- **experimental investigations**
  - stable beam experiments
  - unstable beam experiments
  - direct and indirect approaches
  - experimental techniques & examples

nuclear reaction rates

nuclear reactions in stars: a) produce energy
b) synthesise elements

stars = cooking pots of the Universe

for reaction: $1+2 \rightarrow 3+4$

Total reaction rate: $R_{12} = (1+\delta_{12})^{-1} N_1 N_2 <\sigma v>_{12}$ reactions cm^{-3} s^{-1} N_i = number density

$$\langle \sigma v \rangle = \int \sigma(v)\phi(v)v\,dv$$

Energy production rate: $\varepsilon_{12} = R_{12}\, Q_{12}$ reaction Q-value: $Q=[(m_1+m_2)-(m_3+m_4)]c^2$

Mean lifetime of nucleus 1
against destruction by nucleus 2

$$\tau_2(1) = \frac{1}{N_2 < \sigma v >}$$

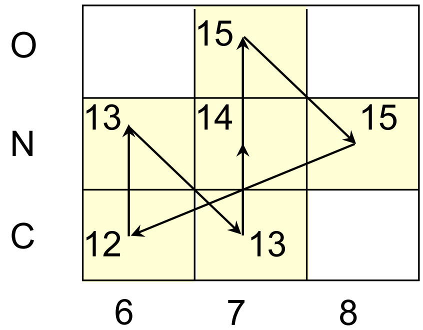
energy production as star evolves ⟸ $<\sigma v>$ = KEY quantity ⟹ change in abundance of nuclei X

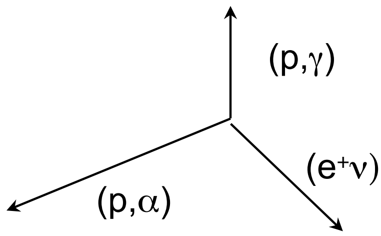
to be determined from experiments and/or theoretical considerations

as star evolves, T changes ⇒ evaluate $<\sigma v>$ for each temperature

NEED ANAYLITICAL EXPRESSION FOR σ!

CNO cycle





$^{12}C(p,\gamma)^{13}N(e^{+}\nu)^{13}C(p,\gamma)^{14}N(p,\gamma)^{15}O(e^{+}\nu)^{15}N(p,\alpha)^{12}C$

cycle limited by β decay of ^{13}N (t ~ 10 min) and ^{15}O (t ~ 2 min)

CNO isotopes act as catalysts

net result: $4p \rightarrow {}^{4}He + 2e^{+} + 2\nu + Q_{eff}$ $\quad Q_{eff} = 26.73$ MeV

nucleosynthesis **energy production**

changes in stellar conditions ⇒ changes in energy production and nucleosynthesis

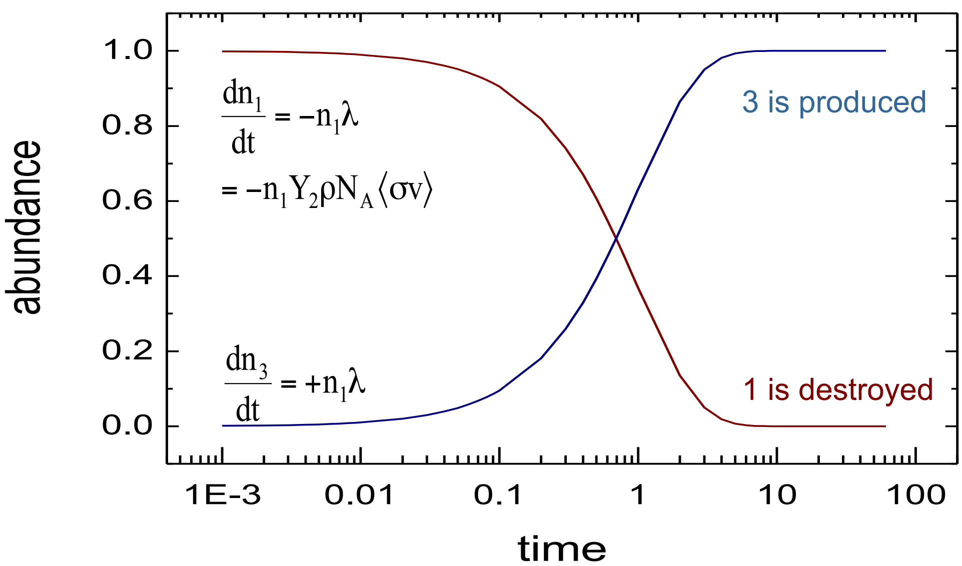
need to know REACTION RATE at all temperatures to determine ENERGY PRODUCTION

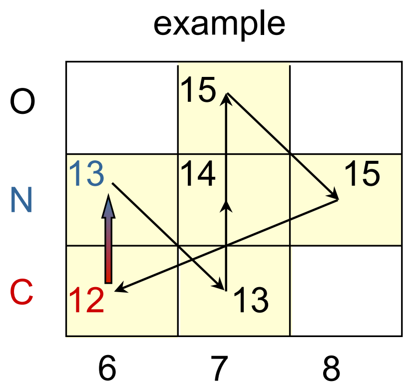
abundance changes and lifetimes

abundances and lifetimes

consider reaction **1+2 → 3**

where **1** is destroyed through capture of **2** and **3** is produced





define:

lifetime of 1 against destruction by with 2:

$$\tau = \frac{1}{\lambda} = \frac{1}{Y_1 \rho N_A \langle \sigma v \rangle}$$

need to know REACTION RATE at all temperatures to determine NUCLEOSYNTHESIS stellar reaction rate $\langle \sigma v \rangle = \int \sigma(v)\phi(v) v dv$

need: a) velocity distribution $\phi(v)$

b) cross section $\sigma(v)$

a) velocity distribution

interacting nuclei in plasma are in thermal equilibrium at temperature T

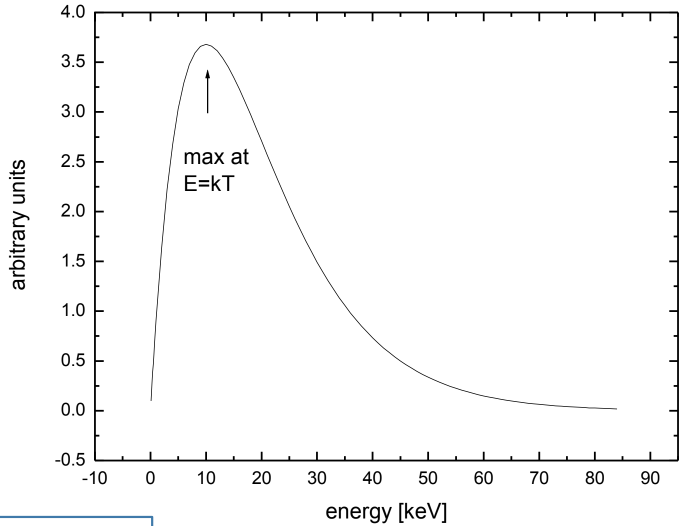
also assume non-degenerate and non-relativistic plasma

$\Rightarrow$ Maxwell-Boltzmann velocity distribution

$$\phi(v) = 4\pi \left(\frac{\mu}{2\pi kT} \right)^{3/2} v^2 \exp\left(-\frac{\mu v^2}{2kT} \right)$$

with $\mu = \frac{m_p m_T}{m_p + m_T}$ reduced mass

v = relative velocity



kT ~ 8.6 x 10^{-8} T[K] keV

example: Sun T ~ 15x10^6 K $\Rightarrow$ kT ~ 1 keV

b) cross section

no nuclear theory available to determine reaction cross section a priori

cross section depends sensitively on:

- the properties of the nuclei involved
- the reaction mechanism

and can vary by orders of magnitude, depending on the interaction

examples:

Reaction	Force	σ (barn)	E_{proj} (MeV)
$^{15}N(p,\alpha)^{12}C$	strong	0.5	2.0
$^{3}He(\alpha,\gamma)^{7}Be$	electromagnetic	10^{-6}	2.0
$p(p,e^{+}\nu)d$	weak	10^{-20}	2.0

1 barn = 10^{-24} cm^2 = 100 fm^2

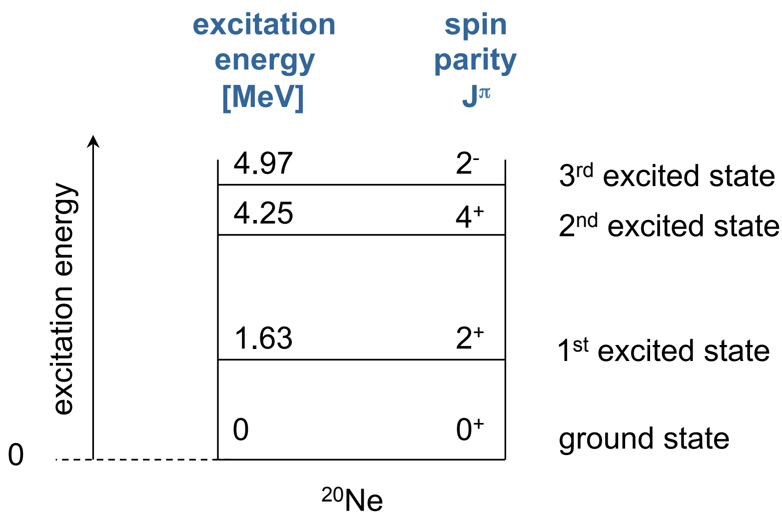
in practice, need **experiments** AND **theory** to determine stellar reaction rates

nuclear properties relevant to reaction rates

recall: nucleons in nuclei arranged in quantised shells of given energy

⇒ nucleus' s configuration as a whole corresponds to discrete energy levels

example



any nucleus in an excited state will eventually decay either by **γ, p, n** or **α emission** with a characteristic **lifetime τ** which corresponds to a **width Γ** in the excitation energy of the state

$$\Gamma = \frac{\hbar}{\tau}$$

Heisenberg' s relationship

the lifetime for each individual exit channel is usually given in terms of **partial widths**

$\Gamma_\gamma, \Gamma_p, \Gamma_n$ and Γ_α with $\Gamma = \sum \Gamma_i$

reaction mechanisms:

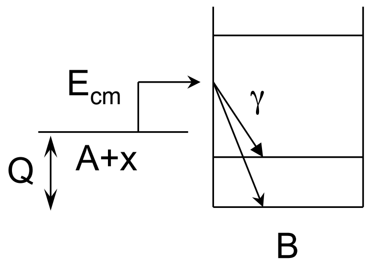
I. **direct (non-resonant) reactions**
II. **resonant reactions**

I. direct (non-resonant) process

one-step process

direct transition into a bound state

example: radiative capture $A(x,\gamma)B$



$$\sigma_\gamma \propto \left| \langle B | H_\gamma | A + x \rangle \right|^2$$

H_γ = electromagnetic operator describing the transition

- reaction cross section proportional to single matrix element
- can occur at all projectile energies
- smooth energy dependence of cross section

other direct processes: stripping, pickup, charge exchange, Coulomb excitation

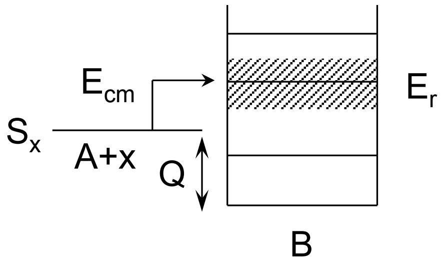
reaction mechanisms

II. resonant process

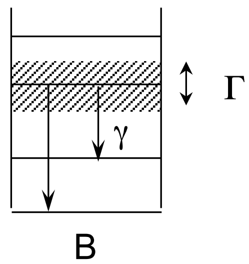
two-step process

example: **resonant radiative capture A(x,γ)B**

1. Compound nucleus formation
(in an unbound state)



2. Compound nucleus decay
(to lower excited states)



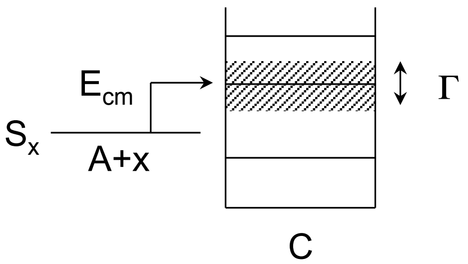
$$\sigma_\gamma \propto \underbrace{\left|\langle E_f | H_\gamma | E_r \rangle\right|^2}_{\substack{\text{compound decay} \\ \text{probability} \propto \Gamma_\gamma}} \; \underbrace{\left|\langle E_r | H_B | A + x \rangle\right|^2}_{\substack{\text{compound formation} \\ \text{probability} \propto \Gamma_x}}$$

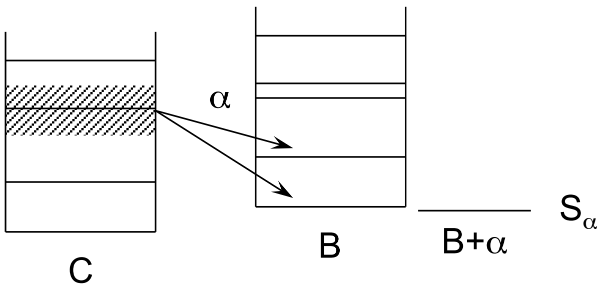
- reaction cross section proportional to two matrix elements
- only occurs at energies $E_{cm} \sim E_r - Q$
- strong energy dependence of cross section

N. B. energy in entrance channel ($Q+E_{cm}$) has to match excitation energy E_r of resonant state, however all excited states have a width ⇒ there is always some cross section through tails example: **resonant reaction A(x,α)B**

1. Compound nucleus formation (in an unbound state)

2. Compound nucleus decay (by particle emission)





$$\sigma_\gamma \propto \underbrace{\left|\langle B+\alpha | H_\alpha | E_r \rangle\right|^2}_{\text{compound decay}} \underbrace{\left|\langle E_r | H_x | A+x \rangle\right|^2}_{\text{compound formation}}$$

compound decay probability $\propto \Gamma_\alpha$ — compound formation probability $\propto \Gamma_x$

N. B. energy in entrance channel (S_x+E_{cm}) has to match excitation energy E_r of resonant state, however all excited states have a width $\Rightarrow$ there is always some cross section through tails

cross section expressions
for direct (non-resonant) reactions

cross sections for direct reactions

example: direct capture $A + x \rightarrow B + \gamma$

$$\sigma = \pi \lambda_x^2 \left| \langle B | H | x + A \rangle \right|^2 P_\ell(E)$$

"geometrical factor"
de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

matrix element
contains nuclear
properties of interaction

probability for barrier penetrability
depends on relative orbital angular
momentum ℓ and energy E

$$\sigma = \frac{1}{E} \cdot S(E) \cdot P_\ell(E)$$

σ = (weak energy dependence) x (strong energy dependence)

S(E) = astrophysical factor contains nuclear physics of reaction
\+ can be easily: graphed, fitted, extrapolated (if needed)

need expression for $P_\ell(E)$

factors affecting transmission probability:

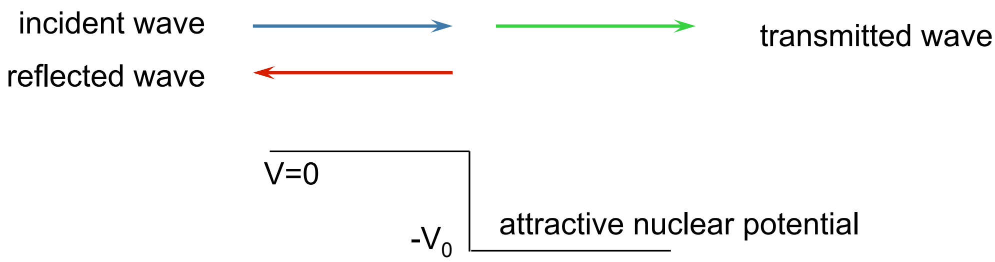
- centrifugal barrier (both for charged particles and neutrons) $V_\ell = \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}$
- Coulomb barrier (for charged particles only)

reactions with neutrons

neutron capture

simplest case: <u>s-wave neutrons</u> $\Rightarrow V_\ell = 0$ and also $V_C = 0$

discontinuity in potential gives rise to partial reflection of incident wave



transmission probability:

$$P_\ell \propto E^{1/2} \quad \text{for } \ell = 0 \quad \text{and hence: } \sigma \propto \frac{1}{\sqrt{E}} = \frac{1}{v}$$

$$P_\ell \propto E^{1/2+\ell} \quad \text{for } \ell \neq 0 \quad \text{and hence: } \sigma \propto E^{\ell-1/2}$$

consequences:

s-wave neutron capture usually dominates at **low energies** (except if hindered by selection rules)

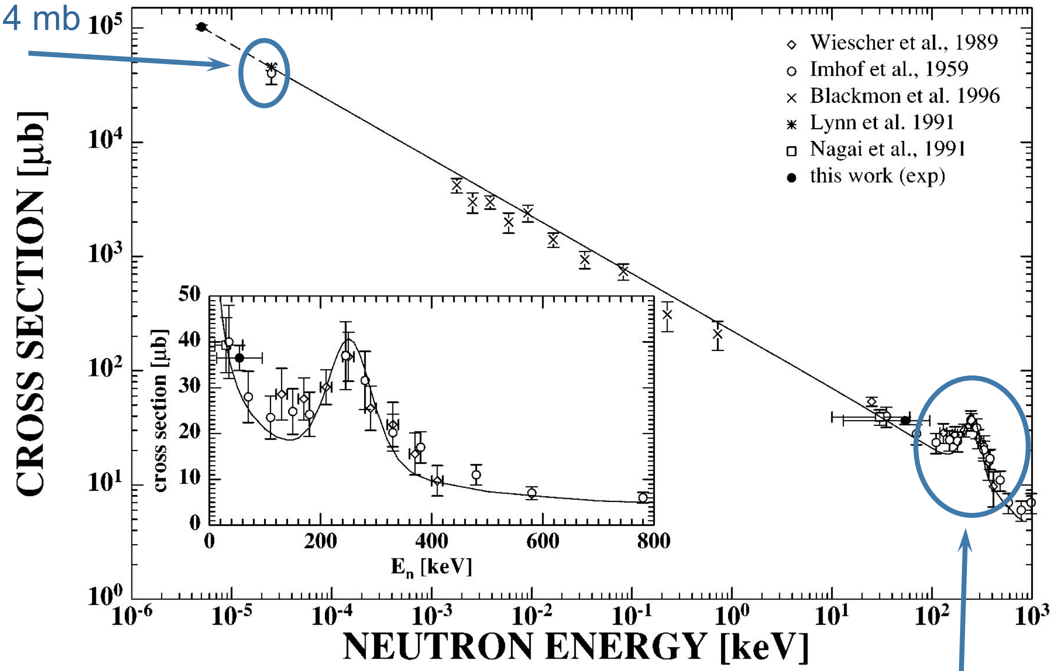
higher ℓ neutron capture only plays role at **higher energies** (or if ℓ=0 capture suppressed)

s-wave neutron capture

$$\sigma \propto \frac{1}{\sqrt{E}} = \frac{1}{v}$$

example: $^{7}Li(n,\gamma)^{8}Li$

thermal
cross section
<σ> = 45.4 mb



deviation from 1/v trend due to
resonant contribution (see later)

$$\langle \sigma v \rangle = \int \sigma(v)\phi(v) v dv = \int \sigma(E) \exp(-E/kT) E dE$$

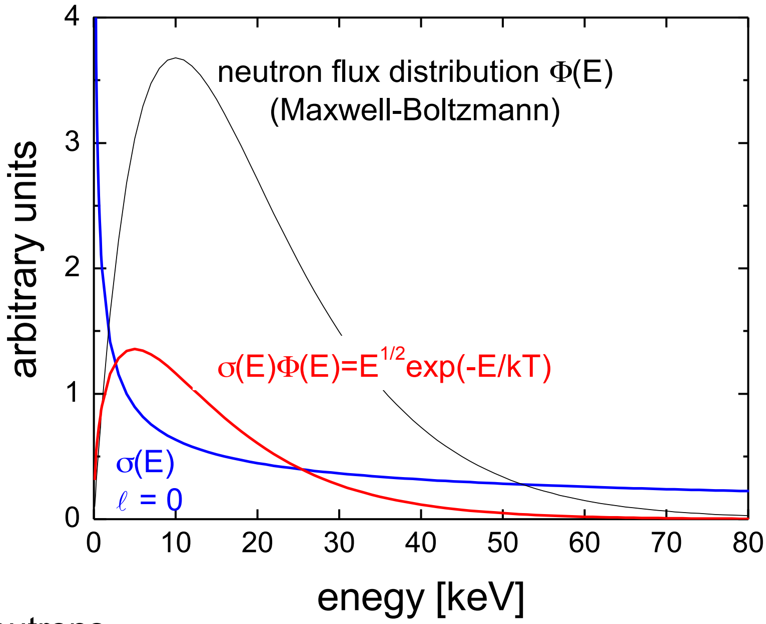
s-wave neutron capture

energy range of interest $E \sim kT$

$\sigma \propto \frac{1}{v}$ ➔ $\sigma v = \text{const} = \langle \sigma v \rangle$

stellar reaction rate

$\langle \sigma v \rangle = v_T \sigma_{th}$



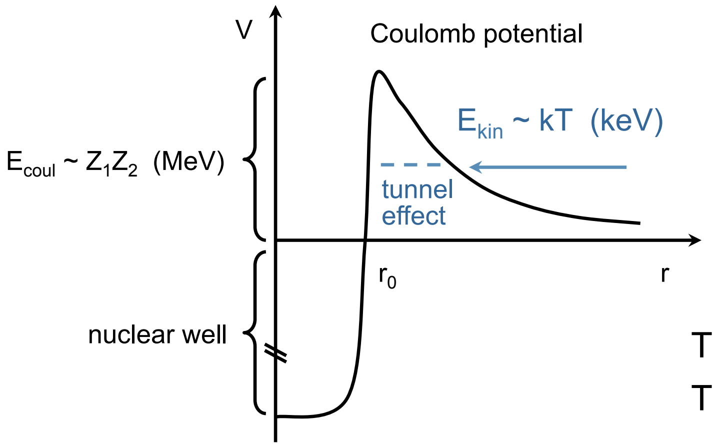
σ_{th} = measured cross section for thermal neutrons

$v_T = \sqrt{\frac{2kT}{\mu}}$ most probable velocity, corresponding to $E_{cm} = kT$

neutron-capture cross sections can be measured DIRECTLY at relevant energies reactions with charged particles

charge-particle reactions

charged particles ⇒ Coulomb barrier



energy available: from thermal motion

$kT \sim 8.6 \times 10^{-8}$ T[K] keV

T ~ 15×10^6 K (e.g. our Sun) ⇒ kT ~ 1 keV
T ~ 10^{10} K (Big Bang) ⇒ kT ~ 2 MeV

during <u>quiescent burnings</u>: $kT \ll E_c$ ⇒ reactions occur through <u>TUNNEL EFFECT</u>

tunneling probability $P \propto \exp(-2\pi\eta)$ $2\pi\eta = 31.29\, Z_1Z_2(\mu/E)^{1/2}$ μ in amu and E_{cm} in keV

$$\sigma = \frac{1}{E}\exp(-2\pi\eta)S(E)$$ for non-resonant reactions

stellar reaction rates for charged particle capture

$$\langle \sigma v \rangle = \int \sigma(v)\phi(v) v dv = \int \sigma(E) \exp(-E/kT) E dE$$

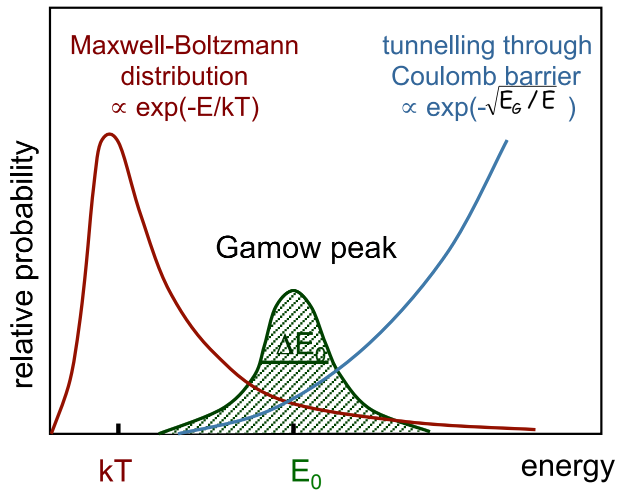
and substituting for σ: $$\langle \sigma v \rangle \propto \int S(E) \exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right) dE$$

maximum reaction rate at E_0: $$\frac{d}{dE}\left[\exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right)\right] = 0$$

Gamow peak

$$E_0 = \left(\frac{bkT}{2}\right)^{3/2} = 0.122\left(Z_1^2 Z_2^2 A\right)^{1/3} T_9^{2/3} \text{ MeV}$$

$$\Delta E = \frac{4}{\sqrt{3}}\sqrt{E_0 kT} = 0.237\left(Z_1^2 Z_2^2 A\right)^{1/6} T_9^{5/6} \text{ MeV}$$



Gamow peak: $E_0 \pm \Delta E_0/2$

energy window of astrophysical interest

$$E_0 = f(Z_1, Z_2, T)$$

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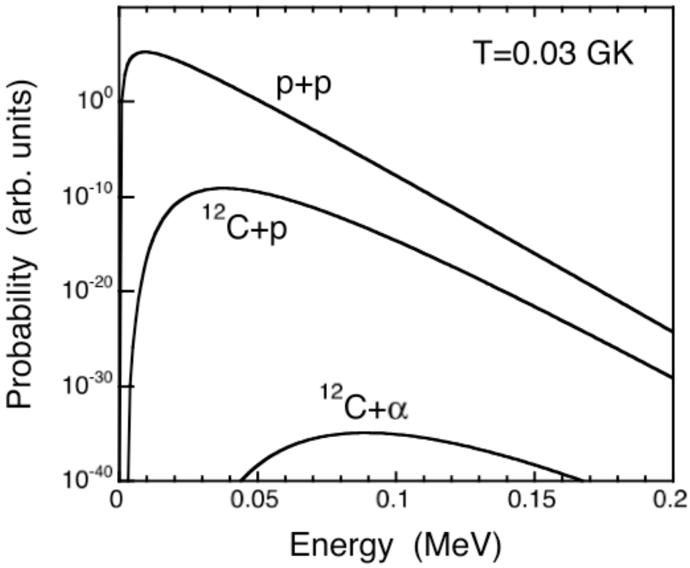
varies depending on reaction and/or temperature



Iliadis, Nuclear Physics of Stars, 2007

Examples: $T \sim 15\text{x}10^6$ K ($T_6 = 15$)

reaction	Coulomb barrier (MeV)	E_0 (keV)	area under Gamow peak ~ <σv>
p + p	0.55	5.9	$7.0\text{x}10^{-6}$
α + ^{12}C	3.43	56	$5.9\text{x}10^{-56}$
^{16}O + ^{16}O	14.07	237	$2.5\text{x}10^{-237}$

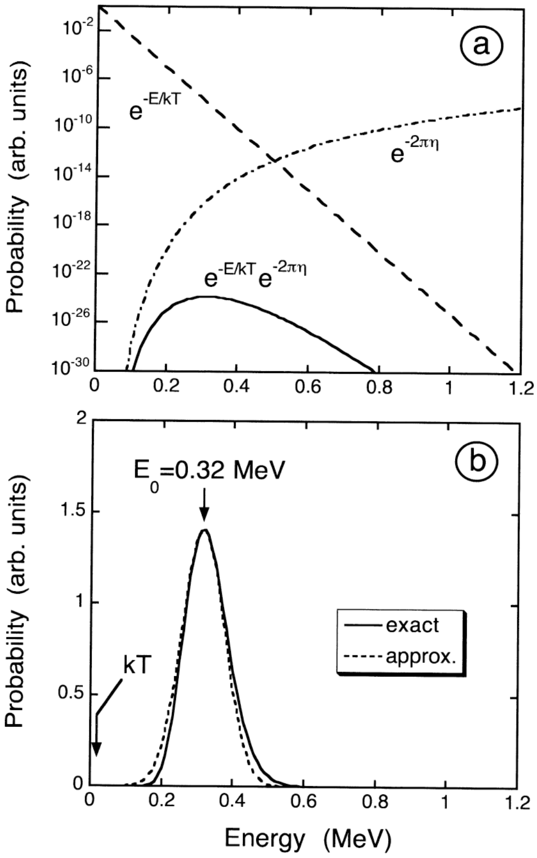
STRONG sensitivity to Coulomb barrier

⇓

separate stages:
- H-burning
- He-burning
- C/O-burning
- …

Gamow peak

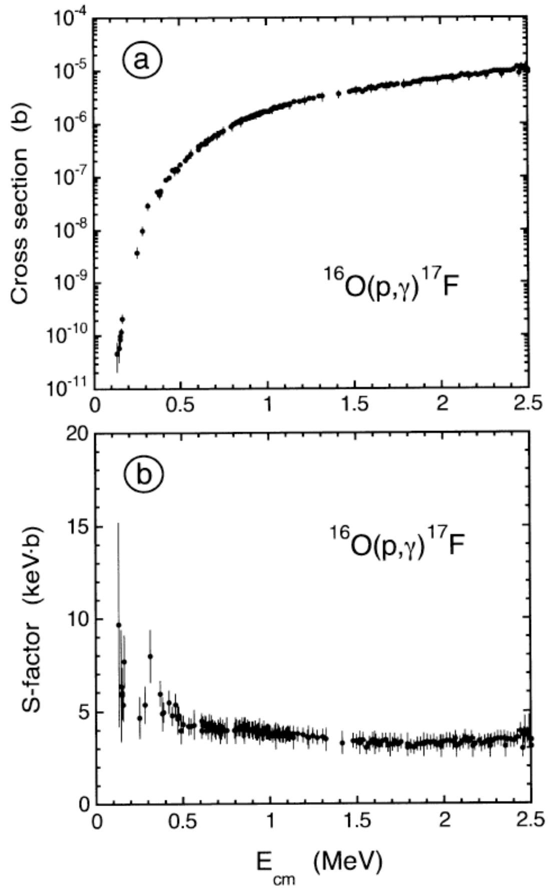
for $^{12}C(\alpha,\gamma)^{16}O$ at T = 0.2GK



Iliadis, Nuclear Physics of Stars, 2007

cross section and astrophysical S-factor

for non-resonant reaction



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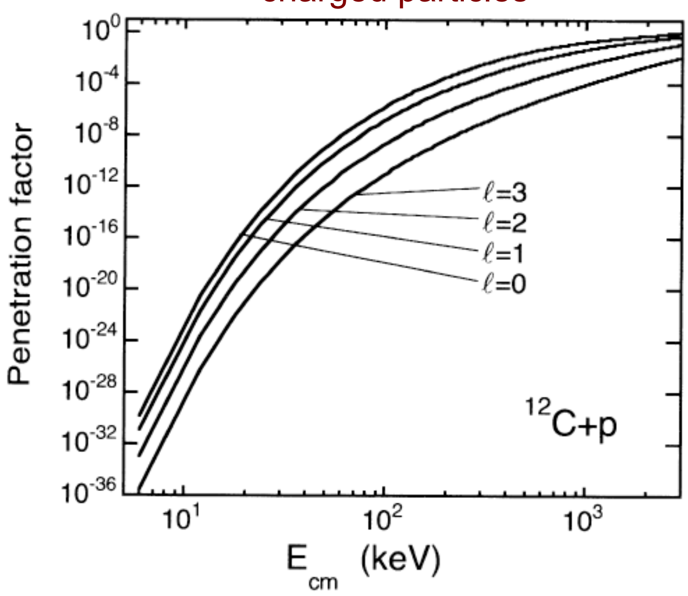
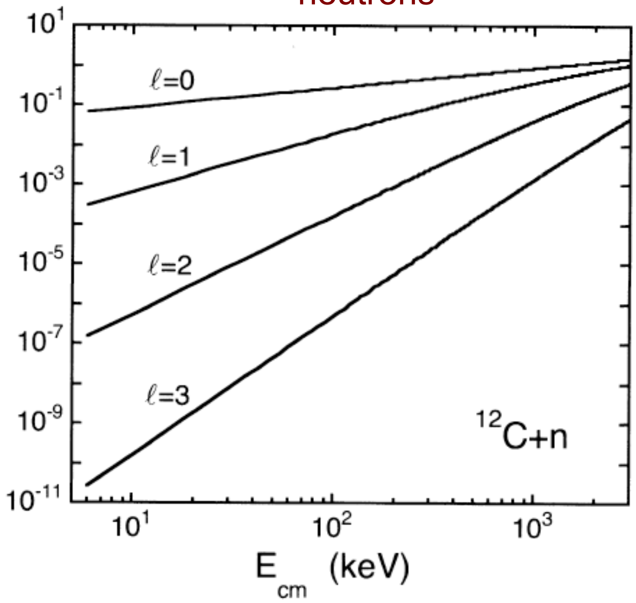
penetrability

charged particles



neutrons



Iliadis, 2007

$$P_\ell \propto \exp\left[-a/\sqrt{E} - b\ell(\ell+1)\right]$$

$$P_\ell \propto E^{1/2+\ell}$$

for $\ell = 0$

$$\sigma = \frac{1}{E}\exp(-2\pi\eta)S(E)$$

$$\sigma \propto \frac{1}{\sqrt{E}} = \frac{1}{v}$$

cross section expressions
for resonant reactions
(neutrons and charged particles)

cross section for resonant reactions

for a single isolated resonance:
resonant cross section given by **Breit-Wigner expression**

$$\sigma(E) = \pi\lambda\!\!\!^-{}^2 \frac{2J+1}{(2J_1+1)(2J_T+1)} \frac{\Gamma_1\Gamma_2}{(E-E_R)^2+(\Gamma/2)^2}$$

for reaction: 1 + T → C → F + 2

geometrical factor
$\propto 1/E$

spin factor ω
J = spin of CN's state
J_1 = spin of projectile
J_T = spin of target

strongly energy-dependent term
Γ_1 = partial width for decay via emission of particle 1
= probability of compound formation via entrance channel

Γ_2 = partial width for decay via emission of particle 2
= probability of compound decay via exit channel

Γ = total width of compound's excited state
= $\Gamma_1 + \Gamma_2 + \Gamma_\gamma + \ldots$

E_R = resonance energy w.r.t. entrance channel threshold

what about penetrability considerations? ⇒ look for energy dependence in partial widths!

partial widths are NOT constant but energy dependent!

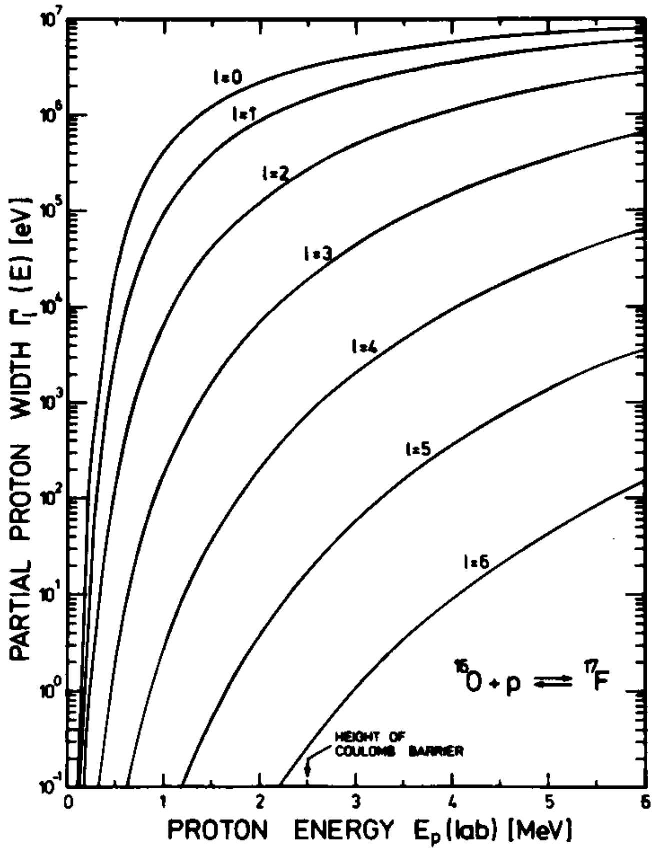
energy dependence of partial widths

particle widths
$$\Gamma_1 = \frac{2\hbar}{R} P_\ell(E_1)\theta_\ell^2$$

θ_ℓ = “reduced width” (contains nuclear physics info)
P_ℓ gives strong energy dependence

example: **$^{16}O(p,\gamma)^{17}F$**

energy dependence of proton partial width Γ_p as function of ℓ



particle partial widths have approximately same energy dependence as penetrability function seen in direct reaction processes

reaction rate for resonant processes

$$\langle \sigma v \rangle = \int \sigma(v)\phi(v) v dv = \int \sigma(E) \exp(-E/kT) E dE$$

$\downarrow$

here Breit-Wigner cross section

$$\sigma(E) = \pi \bar{\lambda}^2 \frac{2J+1}{(2J_1+1)(2J_T+1)} \frac{\Gamma_1 \Gamma_2}{(E-E_R)^2 + (\Gamma/2)^2}$$

integrate over appropriate energy region

E ~ kT	for neutron induced reactions
E ~ Gamow window	for charged particle reactions

if compound nucleus has an exited state (or its wing) in this energy range

$\Rightarrow$ RESONANT contribution to reaction rate (if allowed by selection rules)

typically:

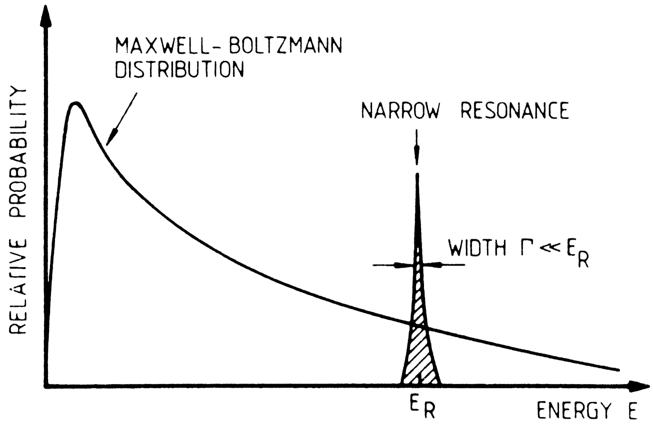
- resonant contribution dominates reaction rate
- reaction rate critically depends on resonant state properties

reaction rate for:

- **narrow resonances**
- broad resonances
- sub-threshold states

narrow resonance case

$\Gamma << E_R$



- resonance must be **near** relevant energy range ΔE_0 to contribute to stellar rate
- MB distribution assumed **constant** over resonance region
- partial widths also **constant**, i.e. $\Gamma_i(E) \cong \Gamma_i(E_R)$

reaction rate for a single narrow resonance

$$\langle \sigma v \rangle_{12} = \left(\frac{2\pi}{\mu_{12} kT} \right)^{3/2} \hbar^2 (\omega\gamma)_R \exp\left(-\frac{E_R}{kT} \right)$$

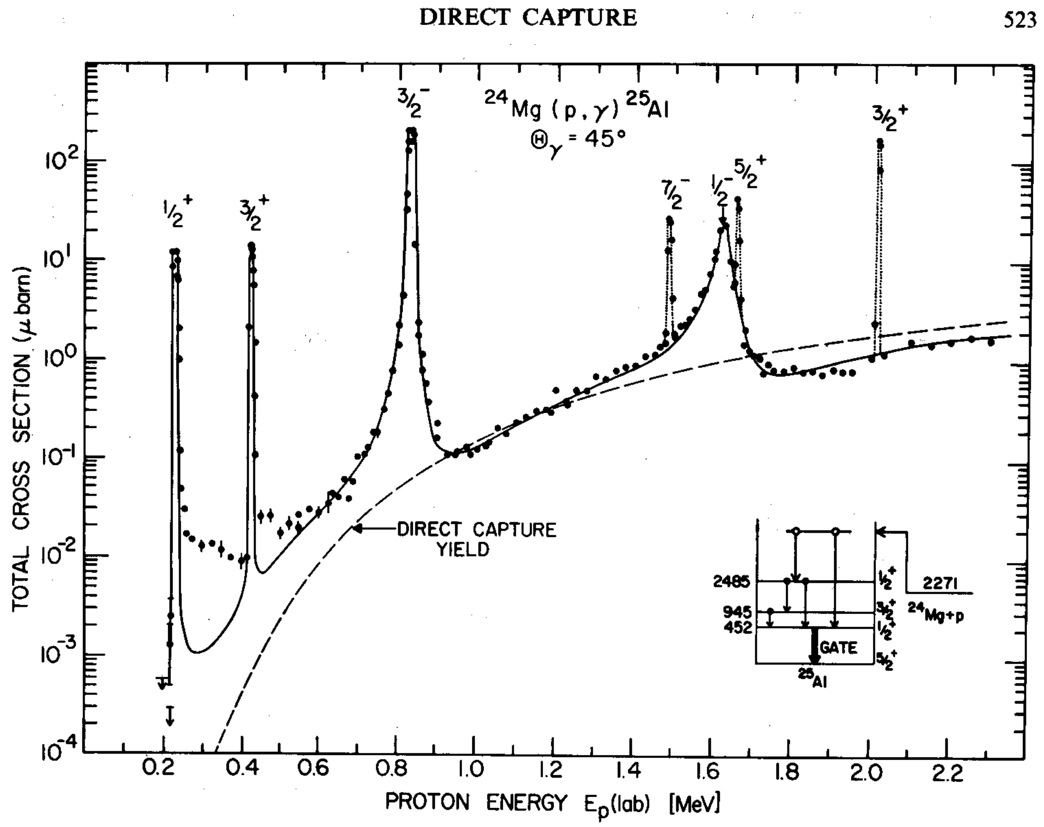
NOTE

exponential dependence on energy means:

- rate strongly dominated by low-energy resonances (**$E_R \rightarrow kT$**) if any
- small uncertainties in E_R (even a few keV) imply large uncertainties in reaction rate

example: $^{24}Mg(p,\gamma)^{25}Al$

the cross section



some considerations…

$$\langle \sigma v \rangle_{12} = \left(\frac{2\pi}{\mu_{12} kT} \right)^{3/2} \hbar^2 (\omega\gamma)_R \exp\left(-\frac{E_R}{kT} \right)$$

rate entirely determined by "**resonance strength**" $\omega\gamma$ and **energy of the resonance** E_R

resonance strength
(= integrated cross section over resonant region)

$$\omega\gamma = \frac{2J+1}{(2J_1+1)(2J_T+1)} \frac{\Gamma_1 \Gamma_2}{\Gamma}$$

(Γ_i values at resonant energies)

often

$$\Gamma = \Gamma_1 + \Gamma_2$$

$$\Gamma_1 << \Gamma_2 \longrightarrow \Gamma \approx \Gamma_2 \longrightarrow \frac{\Gamma_1 \Gamma_2}{\Gamma} \approx \Gamma_1$$

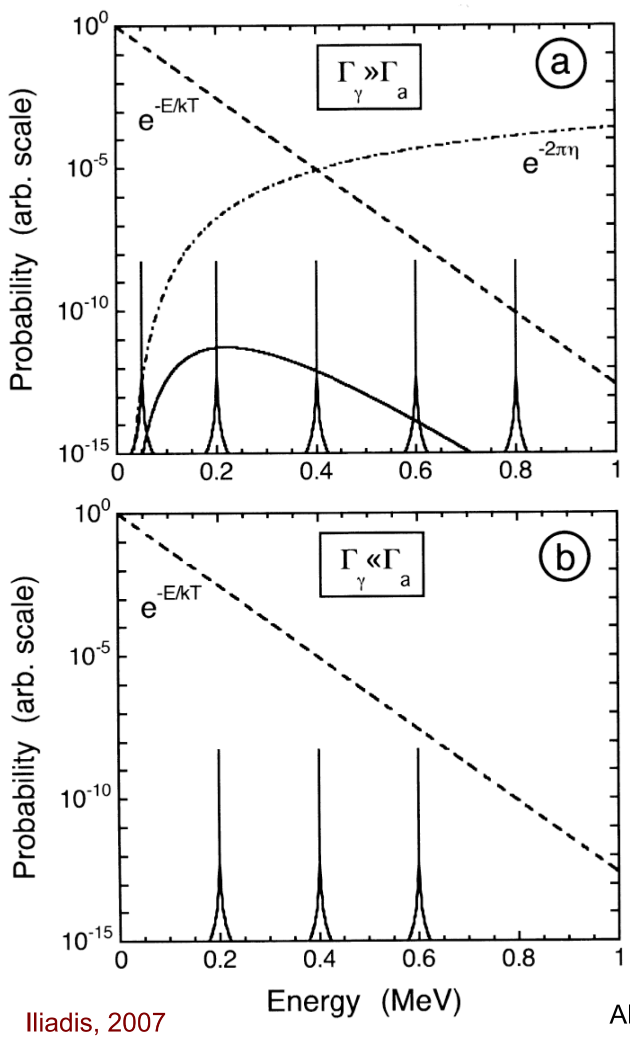
$$\Gamma_2 << \Gamma_1 \longrightarrow \Gamma \approx \Gamma_1 \longrightarrow \frac{\Gamma_1 \Gamma_2}{\Gamma} \approx \Gamma_2$$

reaction rate is determined by the **smaller** width !

experimental info needed:

- partial widths Γ_i
- spin **J**
- energy E_R

note: for many unstable nuclei most of these parameters are **UNKNOWN**!



Iliadis, 2007

resonant strength dominated by particle width

$\omega\gamma = \omega\Gamma_a$ (typically for $E_R \leq 0.5$ MeV)

- strong energy dependence through Coulomb barrier penetration
- only resonances in Gamow window are relevant to reaction rate

resonant strength dominated by gamma width

$\omega\gamma = \omega\Gamma_\gamma$ (typically for $E_R > 0.5$ MeV)

- lowest energies dominate rate because of $\exp(-E_R/kT)$ term
- no Gamow peak exists!
- effect most important at high temperatures

reaction rate through:

- narrow resonances
- **broad resonances**
- sub-threshold states

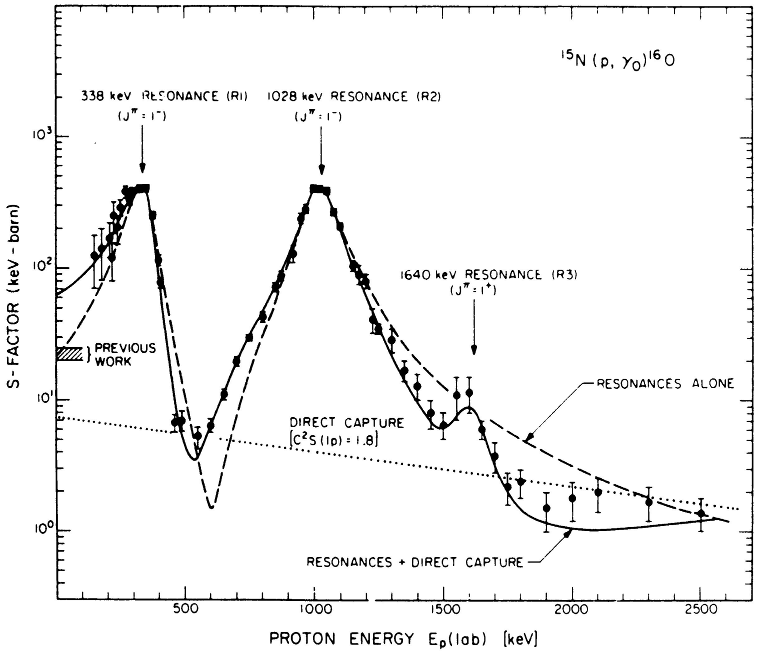
broad resonance case

$\Gamma \sim E_R$

broader than the relevant energy window for the given temperature

resonances outside the energy range can also contribute through their wings

Breit-Wigner formula
+
energy dependence of partial and total widths



assume:

Γ_2= const, Γ = const and use simplified

$$\sigma(E) = \pi \lambda\!\!\!^{-2} \Gamma_1(E)\omega \frac{\Gamma_2}{(E-E_R)^2 + (\Gamma/2)^2}$$

same energy dependence as in direct process

for $E \ll E_R$ very weak energy dependence

N.B. overlapping broad resonances of same $J^\pi \rightarrow$ **interference effects**

3. Sub-threshold resonances

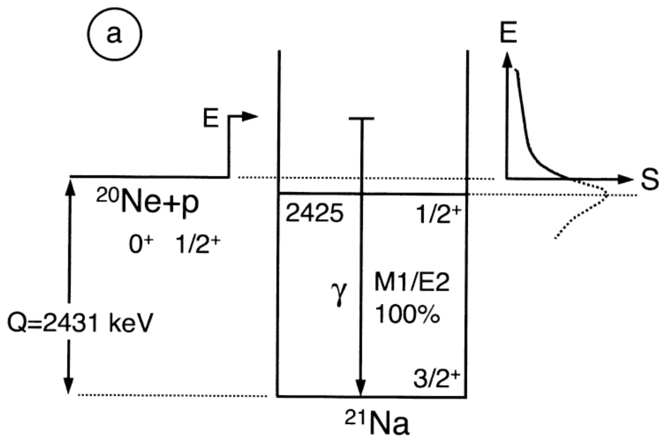
any exited state has a finite width

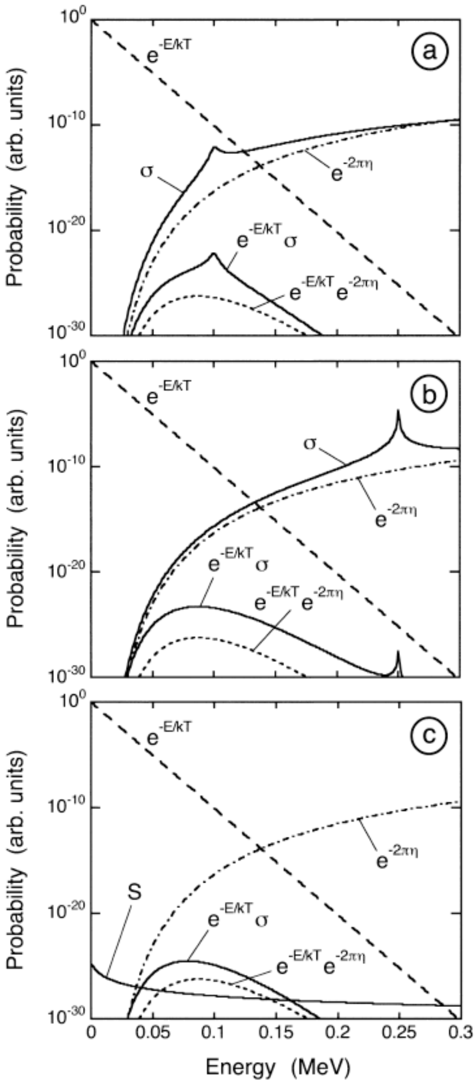
$\Gamma \sim h/\tau$

high energy wing can extend above particle threshold

⇓

cross section can be entirely dominated by contribution of sub-threshold state(s)





broad resonance located within Gamow peak
dominates rate

broad resonance located outside Gamow peak
low-energy wing dominates rate

broad sub-threshold resonance
high-energy wing contributes to rate

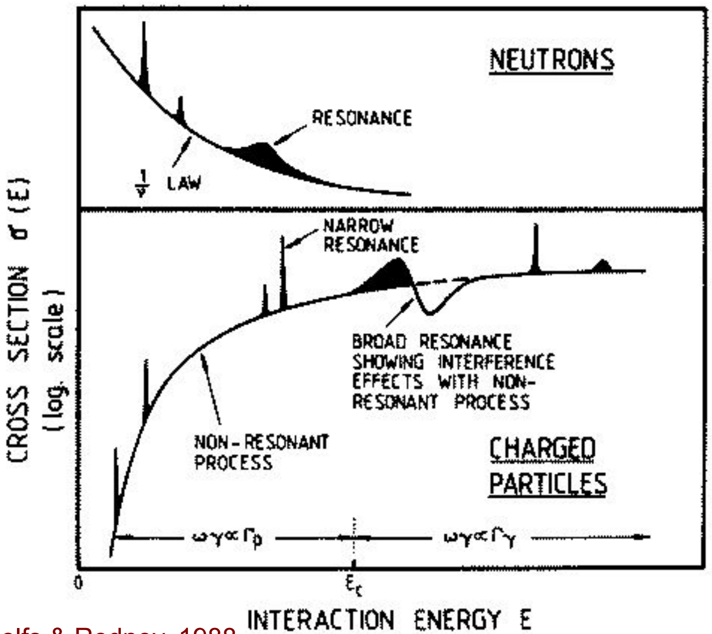
summary

stellar reaction rates include contributions from

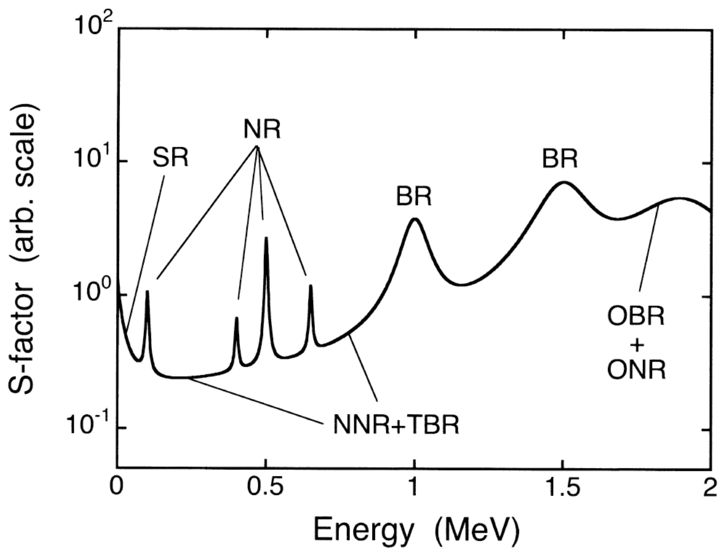
- direct transitions to the various bound states
- all narrow resonances in the relevant energy window
- broad resonances (tails) e.g. from higher lying resonances
- any interference term

total rate

$$\langle \sigma v \rangle = \sum_i \langle \sigma v \rangle_{\mathrm{DCi}} + \sum_i \langle \sigma v \rangle_{\mathrm{Ri}} + \langle \sigma v \rangle_{\mathrm{tails}} + \langle \sigma v \rangle_{\mathrm{int}}$$



Rolfs & Rodney, 1988



Iliadis, 2007

some considerations

resonant and non resonant contributions to stellar reaction rates

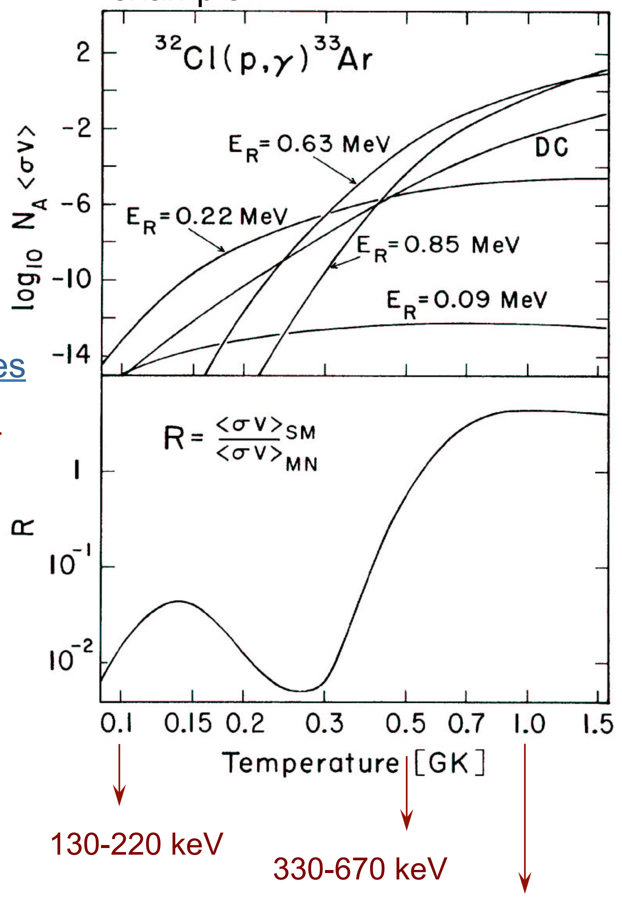
typically

- rates dominated by direct reaction at low temperatures
- rates dominated by resonances at high temperatures

note:

level density in nuclei increases with excitation energy

example



the Gamow window moves to higher energies with increasing temperature

⇒ different resonances play a role at different temperatures

(Coffee) Break