Dense (hadronic and quark) Matter

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Outline:

1. Introduction to nuclear matter

2. The Equation of State

   2. Microscopic vs empirical approach

3. Observational constraints:
   - neutron stars, proto-neutron stars
   - core-collapse supernovae
4. Terrestrial experiments: heavy ion collisions

5. Quark-meson coupling model

6. Summary and outlook

SN 2014J is a type-la supernova in Messier 82 (the 'Cigar Galaxy', M82) discovered in January 2014.
Concept of infinite dense matter:

System of an infinite number of interacting particles in an infinite volume with a finite ratio of a number of particles per unit volume.

No Coulomb force present – no surface effects – translational invariance

Practical use:
interior of neutron stars, core-collapse supernovae, possibly large heavy nuclei

Testing theories under simplified conditions
Phases of dense matter:

**Nuclear matter:** symmetric (equal number of protons and neutrons)

benchmark “magic” numbers for construction of empirical models of high density matter

\[ \rho_0, \frac{E}{A} (\rho_0), S(\rho_0), K_{\infty} \]

Saturation density 0.16 fm\(^{-3}\)
Saturation energy  16 MeV
Symmetry energy  \(\sim 30\) MeV
Incompressibility:  traditional 240+/−30 MeV
NEW VALUE  250 − 315 MeV

**Asymmetric** (unequal number of protons and neutrons)
Pure neutron matter
More generally:

**Hadronic (objects made of quarks) matter:**

Baryons: nucleons, hyperons
Mesons: pion and kaon condensates

**Quark matter:** u-d-s matter and (color) superconducting phases
Structure of high density matter:
Starting: Collins and Perry, PRL 34, 1353 (1975)
Still open questions in 2014:

At what density baryons and mesons will start to loose their identity as bound 3(2)-quark objects?

How would this density compare to the threshold density for creating of hyperons, pions and kaons?

How to incorporate these effects into models?

How can these effects be unambiguously identified in observations?
QCD phase diagram

Temperature $T$ [MeV]

Early universe

Critical point?

Deconfinement and chiral transition

Core collapse supernovae

Net Baryon Density

Hadrons

Quarks and Gluons

MSU

RIKEN

RHIC, LHC

FAIR SIS 300

Neutron stars

Color Superconductor?
The Equation of State (EoS):

Ideal gas:

**Average pressure:**
\[
p = \frac{1}{3} \frac{N}{V} m v^2 \quad \text{N # of molecules of mass } m \text{ in volume } V
\]

**Average molecular kinetic energy:**
\[
\left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3}{2} kT \quad \text{k Boltzmann constant, T temperature}
\]

**Equation of State**
\[
p = \frac{NkT}{V} = \varepsilon(\rho,T)
\]

\(\varepsilon\) total energy density of gas with number density \(\rho = \frac{N}{V}\)
Nuclear matter:

\[ P = \varepsilon(\rho, T) \quad \varepsilon(\rho, T) = \sum_f \left( \frac{E}{A}(\rho, T) \rho \right)_f \quad \mu_B = \frac{(P + \varepsilon)}{\rho} \]

Two key points:

I. The EoS is dependent on composition \text{CONSTITUENTS} + \text{INTERACTIONS}

II. E/A and \text{ITS DENSITY DEPENDENCE} must be determined by nuclear and/or particle models.
Two key points:

The EoS is dependent on composition
CONSTITUENTS + INTERACTIONS

$\varepsilon_f$ and ITS DENSITY AND TEMPERATURE DEPENDENCE

must be determined by nuclear and/or particle models.
Hadronic matter:

Many variants of microscopic and phenomenological models at a different level of complexity:

Mean-field (non)relativistic models

“Ab initio” models with 2- and 3-body forces

Quark-Meson-Coupling model
Quark matter:

MIT bag
Nambu–Jona–Lasinio (NJL)
Polyakov – NJL (PNJL)
Polyakov–Quark Meson (PQM)
Chromo–dielectric (CDM),
Dyson–Schwinger (DS)

Forces (interactions) between the constituents are not known. Each model HAS FREE PARAMETERS which has to fitted to data.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Approx. Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>u up</td>
<td>0.003</td>
<td>2/3</td>
</tr>
<tr>
<td>d down</td>
<td>0.006</td>
<td>−1/3</td>
</tr>
<tr>
<td>charm</td>
<td>1.3</td>
<td>2/3</td>
</tr>
<tr>
<td>s strange</td>
<td>0.1</td>
<td>−1/3</td>
</tr>
<tr>
<td>top</td>
<td>175</td>
<td>2/3</td>
</tr>
<tr>
<td>bottom</td>
<td>4.3</td>
<td>−1/3</td>
</tr>
</tbody>
</table>
Coulomb force:

2 electrical charges:

\[ |F_{Q-q}| = |F_{q-Q}| = k \frac{|q \times Q|}{r^2} \]
Many electrical charges:

principle of superposition

Force acting on a charge $q$ at position $r$ due to $N$ discrete charges:

$$F(r) = \frac{q}{4 \pi \varepsilon_0} \sum_{i=1}^{N} \frac{q_i (r - r_i)}{|r - r_i|^3}$$
Nuclear force

2 nucleons:
- nucleon-nucleon scattering
- tractable with many parameters
- no unique model
Many nucleons: force depends on medium (density) and momentum – strong, weak and elmg interactions play role – intractable?
Pressure in pure neutron matter at sub-saturation density

Whittenbury et al., 2013

Binding energy per particle in symmetric nuclear matter

Li et al., PRC74, 047304 (2006)
Examples of EoS of ud(s) matter in different models

- Chen et al. D-S
- Logoteta et al. CDM
- Logoteta et al. NJL
- Kurkela et al. Perturbative QCD
- Weissenborn et al., MIT bag

Logoteta et al., PRD85, 023003 (2012)
Chen et al., PRD86, 045006 (2012)
Kurkela et al., PRD81, 105021(2010)
Weissenborn et al., 2011
Empirical approach:
Combination of models and observation data

Assumptions: There is only one EoS of high density matter

Questions:

Physical content?

Predictive power?

How sensitive is observation to microphysics?
Do we have enough data to constrain our theories?

Astronomical Observation:

- Neutron stars
- Proto-neutron stars
- Supernovae

Terrestrial Experiments:

- Heavy Ion Collisions
- Hypernuclei
Lattice QCD Thermodynamics:

Calculation currently available only for zero baryo-chemical potential. Extrapolation to finite potential is provided by models – convergence problem.

The \((T,\mu)\) coordinates of the critical point is particularly interesting!

W. Weise / Progress in Particle and Nuclear Physics 67, 299–311 (2012)
Neutron Stars
Extreme conditions in neutron stars allow wide speculations about their internal structure: WHICH ARE REALLY THERE?
Ordinary matter

Electron degenerate matter
Compression 1 ton / cm$^3$

Baryon degenerate matter
Compression 100 million tons / cm$^3$

Atomic nuclei
Electronic orbits
Basic model of (non-rotating) neutron star properties:

Tolman-Oppenheimer-Volkoff (TOV) equations for hydrostatic equilibrium of a spherical object with isotropic mass distribution in general relativity:

\[
\frac{dP}{dr} = - \frac{GM(r)\varepsilon}{r^2} \frac{(1 + P/\varepsilon c^2)(1 + 4\pi r^3 P/M(r)c^2)}{1 - 2GM(r)/rc^2}
\]

\[
M(r) = \int_0^r 4\pi r'^2 \varepsilon(r') \, dr'
\]

**Input:** The Equation of State

\[ P(\varepsilon) \] – pressure as a function of energy density

**Output:** Mass as a function of Radius \[ M(R) \]
Is radius at maximum mass and 1.4 $M_\odot$ a UNIQUE fingerprint of composition?
I. Precise determination of a neutron star mass is not sufficient to compare models with observation.

II. Strong dependence on the equation EoS

III. Do all observed NS have the same EoS and their M and R lie on the same M(R) curve?
A selection of five most accurately measured neutron star masses:

**PSR J0737-3039** the first double pulsar (A,B)
M = 1.249+/-0.001 M⊙ (Lyne et al., Science 303, 1153 (2004))
P = 2.77s (B)

**PSR B1913+16** NS binary (Hulse-Taylor)
P = 59 ms

**PSR J1903+0327** NS on an eccentric orbit around MS star
M=1.667±0.021 M⊙: (Freire, P. C. C. et al., MNRAS, 412, 2763 (2011))
P = 2.5 ms

**PSR J1614-2230** NS+WD
M₉ = 1.97+/−0.04 M⊙ (Demorest at al., Nature 467, 1081 (2010))
P = 3.15 ms

**PSR J0348+0432** NS+WD
M₉ = 2.03+/−0.03 M⊙ (Antoniades et al., Science 340, 448 (2013))
P = 39 ms
Low-mass X-ray binaries inside globular clusters (bursting and transiently accreting)


Steiner, Lattimer and Brown:
1.4 $M_\odot$ 10.4 - 12.9 km 90% conf
arXiv:1305.3242 (May 2013) 90% conf
1.4 $M_\odot$ 11.4 - 12.8 km
1.2 - 2.0 $M_\odot$ 10.9 - 12.7 km


$R_{NS} = 9.1^{+1.3}_{-1.5}$ km (90% confidence) (this work) with all masses

ALL MASSES
The largest central density (Lattimer and Prakash 2005) is constrained above 6.8\times 10^{14}\text{ g/cm}^3 by relatively large systematic errors. As Fig. 3 Neutron star masses does not have measured inclinations (Freire et al. 2007). The most rapidly rotating millisecond pulsar is the object PSR J1748-2021B, with a frequency of 716 Hz, discovered by Hessels et al. (2006). This object spins rapidly enough by recent claim (Kaastra et al. 2007) that \( f > 3\times 10^{14}\text{ s}^{-1} \), implying relatively large masses.

Even very precise information on mass and radius on the same object will not fully solve the uncertainty in the EoS of neutron star matter.
Proto-neutron stars and their evolution
What energy density is available during the formation of the PNS? (essential time up to 60 sec after bounce)

Core collapse supernovae
- Gravitational core collapse of a star with $M > 8 M_\odot$
- Inner core rebounds at $n_b \sim n_w \Rightarrow$ shock wave formation
- Shock wave crosses neutrinospheres $\Rightarrow$ burst of neutrinos
- Hot and dense proto neutron star is left after explosion

Problem: Shock looses too much energy and stalls as standing accretion shock
- $n_SASo$ at $r \sim 10^5 km$

Figures: top: Au Burrows Nature 4343; bottom: Tu Fischers talk at CSQCD II May 2009

T. Fischer, talk at CSQCD II, May 2009
Physical conditions for appearance: hyperons, $\pi$ and $K$ meson condensates, $u$, $d$, $s$ matter +

Threshold densities unknown - strongly model dependent
Can quarks matter be created in NS cores?
Cassiopeia A

Remnant of the historical 1680 SN explosion discovered in 1999 with Chandra X-ray Observatory

Isolated neutron star with a carbon atmosphere and low magnetic field

Precise data on rapid cooling

Rapid cooling is triggered by neutron superfluidity in dense matter enhanced by neutrino emission from the recent onset of the breaking and formation of neutron Cooper pairs in the $^3P_2$ channel in the star’s core. Large proton superconductivity need to be present in the core.

Blaschke et al., PRL 106,081101 (2011)

The cooling rates account for medium-modified one-pion exchange in dense matter and polarization effects in the pair-breaking formations of superfluid neutrons and protons.

Cas A

M = 1.97 $M_\odot$

M = 1.5 $M_\odot$

$\Delta M = 6.0 \times 10^{-8} \Omega_{\odot} \Delta T_\odot = 0.42$

Relativistic model of a 2D rotating neutron star combined with relativistic thermal energy transport: Frequency dependent composition and temperature distribution

Weber, Compstar Tahiti 2012

Negreiros et al., PRD 85, 014019 (2012)
No rapid cooling!!!
Heavy ion collisions
Heavy Ion collisions:

GSI, MSU, Texas A&M, RHIC, LHC
FAIR (GSI), NICA (Dubna, Russia)

**Measurement:**

Beam energy 35 A MeV – 5.5 A TeV
Collisions (Au,Au), (Sn,Sn), (Cu,Cu)
but also (p,p) for a comparison

**Transverse and Elliptical particle flow**

**Calculation:**

Transport models -- empirical mean field potentials
Fit to data → energy density → P (ε) → the EoS
(extrapolation to equilibrium, zero temperature, infinite matter)

Comparison of in-plane and out-of-plane - elliptical flow
Side-ways deflection – transverse flow
Symmetric matter

Transport models with parameters fitted to data on elliptical and transverse flow
Central A-A collision:
Strongly beam energy dependent
Beam energy $< 1$GeV/ A:

Temperature: $< 50$ MeV
Energy density: $\sim 1 - 2$ GeV/fm$^3$
Baryon density $< \rho_0$
Time scale to cool-down: $10^{-22-24}$ s
No neutrinos

\textbf{Strong Interaction:} (S, B and L conserved)
Time scale $10^{-24}$ s

Inelastic NN scatterings,
$N,N^*, \Delta$’s
LOTS of PIONS
strangeness
less important (kaons)

\textbf{Proto-neutron star:}
(progenitor mass dependent)
$\sim 8 - 20$ solar mass

Temperature: $< 50$ MeV
Energy density: $\sim 1$ GeV/fm$^3$
Baryon density $\sim 2-3$ $\rho_s$
Time scale to cool-down: $1 - 10$ s
Neutrino rich matter

\textbf{Strong +Weak Interaction:} (B and L con)
Time scale $10^{-10}$ s

Higher T: strangeness produced in
in weak processes
Lower T: freeze-out

N, strange baryons and mesons,
NO PIONS, leptons

\textbf{? EQUILIBRIUM?}
Observation and experiment does not allow to constrain current theoretical models of high density matter.

Similar situation in low energy nuclear structure.

Try models with parameters constrained by basic physical principles.
QUARK-MESON-COUPLING MODEL

History:
Original: Pierre Guichon (Saclay), Tony Thomas (Adelaide) 1980’
Several variants developed in Japan, Europe, Brazil, Korea, China

Main idea:
Effective model of the MEDIUM EFFECT on baryon structure and interactions
Quark level – coupling between u and d quarks of non-overlapping baryons by meson exchange - significantly simplifies as compared to nucleonic level.
WHAT WE DO:

1. Take a baryon in medium as an MIT bag (with one quion exchange) immersed in a mean scalar field (NJL in progress)

2. Self-consistently include the effects of local couplings of the u and d quarks to a scalar-isoscalar meson (σ) mean field, generated by all the other hadrons in the medium, on the internal structure of that hadron.

3. Calculate the effective mass of the baryon

\[ M_B^* = M_B - w_{σB} g_{σN} \bar{σ} + \frac{d}{2} \bar{w}_{σB} \left( g_{σN} \bar{σ} \right)^2 \]

where \( g_{σN} \) are CALCULATED coupling constants and \( w_{σB} \) are weighting factors allowing using unique σ-N coupling for other baryons. The modification of the internal baryon structure is the only place the quark degrees of freedom enter the model.
4. Construct QMC Lagrangian on a hadronic level in the same way as in RMF but using the effective baryon mass $M^*_B$ and proceed to calculate standard observables.

5. Technically: Full (exchange) Fock term is included (vector and tensor), and $\sigma\omega\rho\pi$ mesons

*(For technical details see Whittenbury et al. arXiv:1307.4166v1*
Parameters (very little maneuvering space):

meson-quark coupling constants:

\[ g_q^q, \ g_q^q, \text{ and } g_q^q \text{ for } q = u, d \ (g_\alpha^s = 0 \text{ for all mesons } \alpha). \]

Fixed to saturation density \(0.16 \text{ fm}^{-3}\), binding energy of SNM -16 MeV and the symmetry energy energy 32.5 MeV

Meson masses: \(\omega, \rho, \pi\) keep their physical values

\(\sigma = 700 \text{ MeV}\)

Cut-off parameter \(\Lambda\) (in form-factors in the exchange terms)

constrained between 0.9 and 1.3 GeV

Free nucleon radius: 1 fm (limited sensitivity within change +/- 20%)

All other parameters either calculated or fixed by symmetry.
WHAT WE GET:

1. Model formulated on quark level which can tackle fundamental issues of nuclear structure within QCD that cannot be addressed by low-energy nuclear theory alone.

2. Scalar polarizability of the baryon:

\[ M_B^* = M_B - g_{\sigma B} \sigma + \frac{d}{2} \left( g_{\sigma B} \sigma \right)^2 \]

Atoms: re-arrangement to oppose the effect of external field – polarization

Nucleons: self-consistent response to the applied mean scalar field tends to oppose that applied field. Increase in the scalar field effectively decreases coupling of the \( \sigma \) to an in-medium baryon \( \rightarrow \) the baryons are source of the scalar field \( \rightarrow \) saturation (equilibrium) will be reached.

NATURAL EXPLANATION FOR SATURATION OF NUCLEAR MATTER
Hyperons


• Derive $\Lambda N$, $\Sigma N$, $\Lambda \Lambda$ effective forces in-medium with no additional free parameters

• Attractive and repulsive forces ($\sigma$ and $\omega$ mean fields) both decrease as # light quarks decreases

• NO $\Sigma$ hypernuclei are bound!

• $\Lambda$ bound by about 30 MeV in nuclear matter (~Pb)

• Nothing known about $\Xi$ hypernuclei – JPARC!
Λ and Ξ hypernuclei in QMC:


Calculation without additional parameters

<table>
<thead>
<tr>
<th></th>
<th>$^8_\Lambda$Yb (Expt.)</th>
<th>$^9_\Lambda$Zr</th>
<th>$^9_\Xi$Zr</th>
<th>$^{208}_\Lambda$Pb (Expt.)</th>
<th>$^{209}_\Lambda$Pb</th>
<th>$^{209}_\Xi$Pb</th>
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<td>$1s_{1/2}$</td>
<td>-22.5</td>
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<td>-7.0</td>
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<td>-24.0</td>
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<td>-17.1</td>
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</table>

Promises Ξ bound by 10 – 15 MeV (to be tested in JPARC)

Increasing split between Λ and Ξ masses with increasing density.
Pressure as a function of energy density as predicted by QMC with hyperons

- Onset of hyperons

- Kinks occur at significant hyperon threshold densities

- The divergence between the standard QMC parameterization and the 'Hartree Only' and "Dirac Only" scenarios highlights the importance of the $\rho_N$ tensor coupling in Hartree–Fock at high density

- The "Nucleon only" BEM EoS is added for a comparison
### Results: Cold neutron star

<table>
<thead>
<tr>
<th>Model</th>
<th>$g_{\sigma N}$</th>
<th>$g_{\omega N}$</th>
<th>$g_{\rho}$</th>
<th>$K_0$</th>
<th>$L$</th>
<th>$R$</th>
<th>$M_{\text{max}}$</th>
<th>$\rho_c^{\text{max}}$</th>
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<tbody>
<tr>
<td>Standard</td>
<td>10.42</td>
<td>11.02</td>
<td>4.55</td>
<td>298</td>
<td>101</td>
<td>12.27</td>
<td>1.93</td>
<td>5.52</td>
</tr>
<tr>
<td>$\Lambda = 1.0$</td>
<td>10.74</td>
<td>11.66</td>
<td>4.68</td>
<td>305</td>
<td>106</td>
<td>12.45</td>
<td>2.00</td>
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<td>12.83</td>
<td>2.14</td>
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<td>124</td>
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<td>2.23</td>
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<td>$R = 0.8$</td>
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<td>12.01</td>
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<td>300</td>
<td>110</td>
<td>12.41</td>
<td>1.98</td>
<td>5.38</td>
</tr>
</tbody>
</table>

Stone, Stone and Moszkowski: accepted in PRC: $250 < K_0 < 315$ MeV
FIG. 9: Gravitational mass versus radius relationship for various scenarios described in the text. The black dots represent maximum mass stars and the colored bars represent observed pulsar constraints.

Antoniadis et al.
Demorest et al.
FIG. 7: (Color online) Species fraction as a function of baryon number density in GBEM, for the standard scenario EoS shown in Fig. 6

QMC predicted composition of HD matter (Y-N potentials calculated)
Figure 1: The composition of neutron star matter as a function of baryon density. Hyperons appear around 2\textit{n}_0. The presence of the \textit{Σ} hyperons depends crucially on the sign of the hyperon-nucleon potential, there are no \textit{Σ} hyperons present for a repulsive potential. Left plot: attractive \textit{Σ} potential, right plot: repulsive \textit{Σ} potential (see [13] for the details of the model used).

Modern many-body approaches use as input the two-body potentials as deduced from hyperon-nucleon scattering data. For the Nijmegen soft-core hyperon nucleon potentials Vidana et al. find that the maximum mass is only $M_\text{max} = 1.47 M_\odot$ which reduces to even $M_\text{max} = 1.34 M_\odot$ when the hyperon-hyperon potentials are switched on [19]. Also Baldo et al. find values of $M_\text{max} = 1.26 M_\odot$ even when including three-body nucleon interactions [18]. More recently Schulze et al. [20] and Djapo et al. [26] confirm that $M_\text{max} < 1.4 M_\odot$ for modern microscopic (ab initio) approaches. Hence, the neutron star equation of state gets too soft at high densities giving too low masses. Probably the underlying reason are missing three-body forces for hyperons (YNN, YYN, YYY), which give additional repulsive contributions at high densities. If so then it seems that neutron stars can not live without hyperon three-body forces. Certainly, here more input is needed from hypernuclear physics by e.g. the study of light double hypernuclei in the near future to extract the hyperonic three-body forces.

5. Maximum possible mass of neutron stars

There is another strange hadron with strong relations to the physics of the maximum possible mass of neutron stars. Kaons produced subthreshold in heavy-ion experiments can serve as a messenger of the high-density zone created in the collision. Kaons are produced by associated production e.g. via $\text{NN} \rightarrow \text{nΛK}$, and $\text{NN} \rightarrow \text{NNK}K$ in elementary proton-proton collisions. In the medium, i.e. in heavy-ion collisions, rescattering processes open up as $\text{πN} \rightarrow \text{ΛK}$, $\text{πΛ} \rightarrow \text{N}K$ from produced pions which have a lower q-value and are therefore able to pump up the kaon production rates substantially compared to the elementary pp-collisions. At subthreshold bombarding energies of heavy ions the matter can be compressed up to 3\textit{n}_0. However, kaons also have a mean free path, they scatter lastically within nucleons and pions, only hyperons can absorb them as kaons carry an antistrange quark. Hence, kaons can escape from the high density zone J. Schaffner-Bielich / Nuclear Physics A 835 (2010) 279–286

RMF with GM1 interaction empirical Y-N potentials fitted selfconsistently to data

Application to finite nuclei:

Guichon, Matevosyan, Sandulescu, Thomas, NPA 772, 1, 2006

Density dependent force in a non-relativistic approximation can be derived from QMC. The Hamiltonian depends on QMC coupling constants and polarizability but has formally similar structure to the Skyrme forces.

\[
\mathcal{H}_0 + \mathcal{H}_3 = \rho^2 \left[ \frac{-3 G_\rho}{32} + \frac{G_\sigma}{8 (1 + d \rho G_\sigma)^3} - \frac{G_\sigma}{2 (1 + d \rho G_\sigma)} + \frac{3 G_\omega}{8} \right] +
\]

\[
(r_n - r_p)^2 \left[ \frac{5 G_\rho}{32} + \frac{G_\sigma}{8 (1 + d \rho G_\sigma)^3} - \frac{G_\omega}{8} \right],
\]

highlights scalar polarizability
with the weak isospin dependence employed in the relativistic mean field models, in which the contribution of Fock (exchange) terms is neglected [22].

Starting from the QMC energy functional one can easily derive the corresponding Hartree–Fock (HF) equations. They have a form similar to the Skyrme–HF equations, apart from the rearrangement term and the one-body spin–orbit interaction, which (as discussed above) have a different density and isospin dependence. The HF equations were solved in coordinate space, following the method described in Ref. [12] and the Coulomb interaction was treated in a standard way—i.e., the contribution of its exchange part was calculated in the Slater approximation. The calculations were performed for the doubly magic nuclei $^{16}$O, $^{40}$Ca, $^{48}$Ca and $^{208}$Pb. For definiteness, the $\sigma$ meson mass has been set to $m_{\sigma} = 700$ MeV, as suggested by the comparison with the SkM$^*$ interaction. At this point we recall that the QMC model is essentially classical because both the position and velocity of the bag are assumed known in the energy expression (3). The quantization then leaves some arbitrariness in the ordering of the momentum dependent pieces of the interaction. As pointed out in previous work [11], in the non-relativistic approximation the difference between the orderings is equivalent to a change of about 10% in $m_{\sigma}$. In this work the ordering is fixed by the relativistic expression chosen for the operator $K$, Eq. (17). Then the non-relativistic reduction then leads to an ordering which is not the same as in Ref. [11]. This is why the $\sigma$ meson mass that we use here is somewhat higher. Note that this ordering ambiguity is only of concern in the case of finite nuclei. In uniform matter, which is the relevant approximation for neutron stars, the problem does not exist.

The results for the binding energies and charge radii are shown in Table 3. The charge densities are calculated with the proton form factor usually employed in the Skyrme–HF calculations [12]. From Table 3 one can see that QMC-HF gives results which are in reasonable agreement with the experimental values. The agreement is not as good as that given by the recent Skyrme or RMF models, but one should keep in mind that in these models the experimental values for the binding energies and radii are included in the fitting procedure, which is not the case for the QMC functional.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>$E_B$ (MeV, exp)</th>
<th>$E_B$ (MeV, QMC)</th>
<th>$r_c$ (fm, exp)</th>
<th>$r_c$ (fm, QMC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O</td>
<td>7.976</td>
<td>7.618</td>
<td>2.73</td>
<td>2.702</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>8.551</td>
<td>8.213</td>
<td>3.485</td>
<td>3.415</td>
</tr>
<tr>
<td>$^{48}$Ca</td>
<td>8.666</td>
<td>8.343</td>
<td>3.484</td>
<td>3.468</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>7.867</td>
<td>7.515</td>
<td>5.5</td>
<td>5.42</td>
</tr>
</tbody>
</table>

Table 4

Comparison between the QMC and “experimental” spin–orbit splittings. Because the experimental splittings are not so well known in the case of $^{48}$Ca and $^{208}$Pb, we give the values corresponding to the Skyrme Sly4 prediction.

<table>
<thead>
<tr>
<th></th>
<th>Neutrons (exp)</th>
<th>Neutrons (QMC)</th>
<th>Protons (exp)</th>
<th>Protons (QMC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O, 1p$<em>{1/2}$–1p$</em>{3/2}$</td>
<td>6.10</td>
<td>6.01</td>
<td>6.3</td>
<td>5.9</td>
</tr>
<tr>
<td>$^{40}$Ca, 1d$<em>{3/2}$–1d$</em>{5/2}$</td>
<td>6.15</td>
<td>6.41</td>
<td>6.00</td>
<td>6.24</td>
</tr>
<tr>
<td>$^{48}$Ca, 1d$<em>{3/2}$–1d$</em>{5/2}$</td>
<td>6.05 (Sly4)</td>
<td>5.64</td>
<td>6.06 (Sly4)</td>
<td>5.59</td>
</tr>
<tr>
<td>$^{208}$Pb, 2d$<em>{3/2}$–2d$</em>{5/2}$</td>
<td>2.15 (Sly4)</td>
<td>2.04</td>
<td>1.87 (Sly4)</td>
<td>1.74</td>
</tr>
</tbody>
</table>
expect different values for nuclei with large isospin asymmetry. However, as can be seen from Table 4, the differences are rather small. This is primarily because the spin–orbit splitting depends on the product of the spin–orbit form factor and the corresponding single-particle wave functions. Thus, if the wave functions are not strongly localised in the surface region, where $W_\tau(r)$ is effective, the influence of the isospin dependence of $W_\tau(r)$ upon the splitting need not be so significant.

In Figs. 1, 2 we show the proton and neutron densities calculated with the QMC model and with the Sly4 Skyrme force [19]. In the proton case we also show the experimental values [23].

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**QMC proton density distribution compared with experiment and Skyrme SLy4**
QMC has a natural explanation for saturation of nuclear matter and in-medium effects through many-body forces

It is not limited to nucleons but can be applied to hyperons and calculate interaction of any hadron in nuclear medium with NO ADDITIONAL parameters.

Yields effective, density dependent $\Lambda N$, $\Sigma N$, $\Xi N$ forces (not yet published) with NO additional parameters – reproduces known properties of hypernuclei

Can be used to derive density-dependent effective force such as the Skyrme force which performs well in finite nuclei (HF+BCS QMC code for axially symmetric nuclei has been just developed and is in a testing stage (with P. - G. Reinhard)

BUT
IF QMC is as valid as we believe, it has to yield predictions consistent with results in other areas of nuclear physics and astrophysics.

FUTURE: EoS for supernova matter (Chikako Ishizuka, Akira Ohnishi) (QMC at finite temperature)
Statistical analysis of mass and radii of NS (Andrew Steiner)
Projected shell model (Yang Sun in Shanghai)
Ab-initio calculation of light nuclei (Emiko Hiyama at RIKEN)
Rotating neutron stars (Fridolin Weber + collaborators)

+ + +

SUGGESTIONS WELCOME
SUMMARY

I. We do not understand behaviour of hadrons in dense medium.

II. Current models have limited predictive power – they have too many parameters and it is impossible to constrain them unambiguously.

III. Models are often adjusted to fit only a selected class of data well, but their failure elsewhere is neglected. Such models cannot be right. Even “minimal” models are of limited use in a broader context.

POSSIBLE SOLUTION?

Evaluate basic assumptions of each model and regions of applicability.

Focus on models with INDIVIDUAL parameters constrained by physics.

Microphysics is important!

DATA LIMITED BY AVAILABLE TECHNIQUE – PHYSICS SHOULD BE ADOPTED AS A CONSTRAINT.
N, Λ, Ξ, ω, D, J/Ψ in nuclear matter

QCD & hadron structure

Density dependent effective NN (and N Λ, N Ξ) forces

Structure of finite nuclei & hypernuclei

n star

∞ nuclear matter

quark matter

Courtesy Anthony Thomas University of Adelaide
Back-up slides
Pressure in pure neutron matter as calculated in different models
Left panel: without 3BF \hspace{1cm} Right panel: the same but with 3BF.
DBHF added in right panel [Tsang et al., PRC 86, 015803 (2012)]
Symmetry energy $S$ (top) and its slope $L$ (bottom) as a function of baryon number density as calculated in QMC.

Effect of the Fock term:

**Standard:** vector + tensor

**Dirac:** vector

**Hartree:** no Fock term

$\Lambda$ cut-off parameter of the form-factor in the Fock term.
Pure neutron matter energy per particle as a function of density as obtained in QMC, in comparison with complete CEFT at N$^3$LO order for more details of the latter see: I. Tews, T. Krueger, K. Hebeler and A. Schwenk, Phys. Rev. Lett. 110 (2013) 032504
Updated constraints Tsang et al., PRC 86, 015803 (2012)

\[ S(\rho) = \frac{1}{2} \frac{\partial^2 E}{\partial \beta^2} \bigg|_{\rho, \beta = 0}, \]

\[ E = \frac{\epsilon_{\text{hadronic}}}{\rho}, \]

\[ \mathcal{E} = \frac{1}{\rho} \left( \epsilon_{\text{hadronic}} - \sum_B M_B \rho_B \right). \]